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A recursive subsystem synthesis method for repeated closed loop structure in multibody dynamics †

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Abstract

A recursive subsystem synthesis method has been proposed for efficient analysis for a repeated closed loop structure in multibody dynamics. Virtual work form of equations of motion has been used to reduce subsystem equations of motion. Position, velocity, and acceleration analyses are carried out subsystem by subsystem in the forward recursive fashion. Effective mass matrices and force vectors are computed subsystem by subsystem in the backward recursive fashion. An excavator example is used for a repeated closed loop structure to validate the proposed method.

Keywords: Multibody dynamics; Recursive subsystem synthesis method; Repeated closed loop structure

1. Introduction

UV-Nanoimprint Lithography (UN-NIL) has been Recently, efficient computation or real-time computation of multibody systems has gained attention to the hardware-in the loop simulations or to the operator-in the loop simulation with virtual reality. The subsystem synthesis method is one of the efficient methods for real-time computations [1]. In the subsystem synthesis method, equations of motion for subsystems are formed separately and base body equations of motion are constructed separately. Thus, instead of solving a large system of equations of motion for entire systems, several small equations of motion for subsystems are solved. This enables the subsystem synthesis method to be efficient compared with the conventional method. The subsystem synthesis method is very effective; when the system contains topologically several identical subsystems, one subsystem module can be reused several times in the program point of view. So far, the subsystem synthesis method has been only applied to a vehicle system that contains identical independent suspension subsystems [1-3]. The vehicle system has special topology, the so-called "star shape" as shown in Fig. 1. In this topology, dynamic effects of the each subsystem are computed as the form of an effective mass matrix and an effective force vector, and added into the chassis's equations of motion.

However, the subsystem synthesis method has not been applied to the system that contains repeated closed loop structure as shown in Fig. 2. In this paper, the recursive subsystem synthesis method is proposed for the efficient computation for such a topological system.

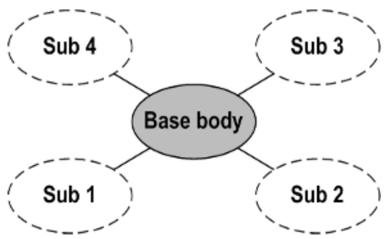

Fig. 1. Star shape topology of MBS.

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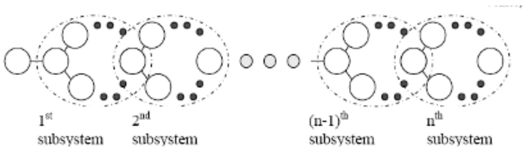

Fig. 2. Repeated subsystem topology.

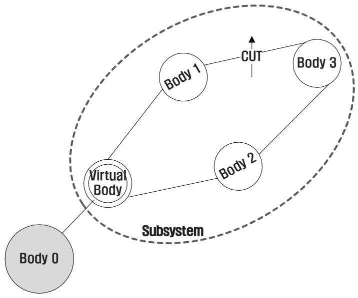

Fig. 3. A typical subsystem topology.

2. Recursive subsystem synthesis method

2.1 Reduction procedure in the subsystem synthesis method

In this section, a reduction procedure is explained with one subsystem which consists of several bodies and joints as shown in Fig. 3. The virtual body is defined as the reference body of the subsystem and it does not have any inertia and force quantities. The virtual body is connected to the base body (Body 0) by a fixed joint.

The equations of motion of this subsystem are expressed in virtual work form as Eq. (1).

$$\boldsymbol{\delta}\hat{\mathbf{z}}_0^{\mathrm{T}}(\hat{\mathbf{M}}_0\dot{\hat{\mathbf{y}}}_0 - \hat{\mathbf{Q}}_0) + \left(\sum_{\mathrm{i}=0}^{\mathrm{nb\_sub}} \boldsymbol{\delta}\hat{\mathbf{z}}_{\mathrm{i}}^{\mathrm{T}}(\hat{\mathbf{M}}_{\mathrm{i}}\dot{\hat{\mathbf{y}}}_{\mathrm{i}} - \hat{\mathbf{Q}}_{\mathrm{i}})\right)_{\mathrm{sub}} = \mathbf{0} \quad (1)$$

Using the relationships between the joint and the Cartesian coordinate system [5], Lagrange multiplier theorem [6, 7] and the generalized coordinate partitioning method [7], the subsystem equations of motion can be reduced as Eq. (2).

$$\boldsymbol{\delta}\hat{\mathbf{z}}_0^{\mathrm{T}}(\hat{\mathbf{M}}_0\dot{\hat{\mathbf{y}}}_0 - \hat{\mathbf{Q}}_0) + \left(\boldsymbol{\delta}\hat{\mathbf{z}}_0^{\mathrm{T}}(\breve{\mathbf{M}}^{\mathrm{c}}\dot{\hat{\mathbf{y}}}_0 - \breve{\mathbf{P}}^{\mathrm{c}})\right)_{\mathrm{Sub}} = \mathbf{0} \quad (2)$$

where, $\breve{\mathbf{M}}^{\mathrm{c}} = \bar{\mathbf{M}}_{\mathrm{yy}} - \mathbf{M}_{\mathrm{y}}^{*\mathrm{T}}(\mathbf{M}^*)^{-1}\mathbf{M}_{\mathrm{y}}^*$ and $\breve{\mathbf{P}}^{\mathrm{c}} = \mathbf{Q}_{\mathrm{y}}^* - \mathbf{M}_{\mathrm{y}}^{*\mathrm{T}}(\mathbf{M}^*)^{-1}\mathbf{Q}_{\mathrm{q}}^*$. $\breve{\mathbf{M}}^{\mathrm{c}}$ is called an effective mass matrix and $\breve{\mathbf{P}}^{\mathrm{c}}$ is called an effective force vector [5]. From Eq. (2), the equation of motion for the base body 0 can be obtained as,

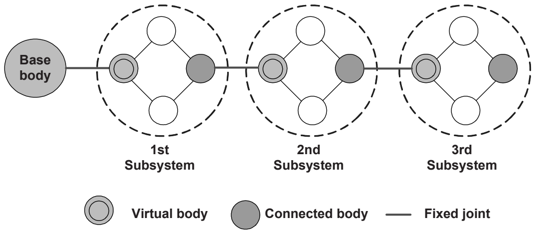

Fig. 4. MBS with repeated closed loop structure.

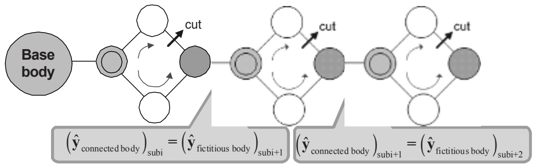

Fig. 5. Forward subsystem based recursive computation.

$$(\hat{\mathbf{M}}_0 + \breve{\mathbf{M}}^{\mathrm{c}})\dot{\hat{\mathbf{y}}}_0 = \hat{\mathbf{Q}}_0 + \breve{\mathbf{P}}^{\mathrm{c}} \quad (3)$$

2.2 Recursive subsystem synthesis method for the repeated topology of multibody systems

To apply the reduction procedure to the multibody system as shown in Fig. 2, the system is divided into several subsystems as shown in Fig. 4. The connected body in the subsystem is the body that is connected to the virtual body of the other subsystem by the fixed joint. Thus, the kinematic properties such as position, velocity and acceleration of the connected body are the same as those of the virtual body.

Position and velocity of the subsystem can be obtained by solving loop closure constraint equations. Then positions and velocities of all the bodies, including the connected body, and joints within the subsystem can be computed. As described earlier, the connected body motion is the same as the virtual body motion of the outboard subsystem, because they are connected to the fixed joint as shown in Fig. 5. Thus, the position and velocity analyses can be done from the first subsystem to the 3rd subsystem in the forward recursive fashion.

After position and velocity analyses have been performed, the effective mass matrix and the effective force vector of each subsystem can be calculated repeatedly by using the reduction procedure from the outboard subsystem (the 3rd subsystem) to the inboard subsystem (the 1st subsystem) in the backward recursive fashion, as shown in Fig. 6. In this procedure, the effective mass matrix and force vector from the outboard subsystem are added to the mass matrix and force vector of the connected body in the adjacent inboard subsystem.

If this reduction procedure is carried out all the way to the base body, then Eq. (4) can be obtained finally as

$$\delta\hat{\mathbf{z}}_0^{T}\left(\left(\hat{\mathbf{M}}_0+\breve{\mathbf{M}}^{c}\right)\dot{\hat{\mathbf{y}}}_0-\left(\hat{\mathbf{Q}}_0+\breve{\mathbf{P}}^{c}\right)\right)=\mathbf{0} \tag{4}$$

After solving Eq. (4) for the base body acceleration, joint acceleration and Cartesian acceleration can be also calculated from the inboard subsystem to the outboard subsystem in a similar fashion to the position and velocity analysis.

3. Excavator simulation

3.1 Excavator modeling

To validate the proposed method, an excavator manipulator arm has been modeled as shown in Fig. 7.

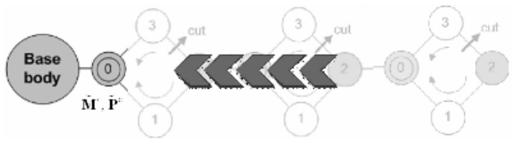

Fig. 6. Apply the reduction procedure by subsystems.

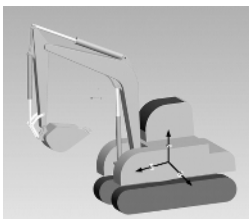

Fig. 7. Excavator model.

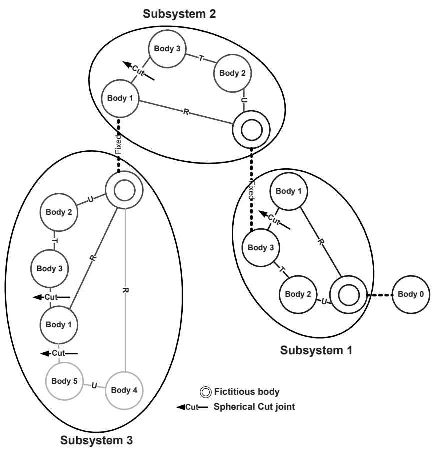

Fig. 8. Subsystem definition for the of excavator topology.

To apply the proposed recursive subsystem synthesis method to the excavator topology, subsystems are defined as shown in Fig. 8.

3.2 Analytical method

One of merits of the subsystem synthesis method is that the different formulation can be applied in the subsystems as long as the interface variables, such as Cartesian position, velocity, accelerations of the connected body and effective mass matrix and effective force vector of the virtual body in the outboard subsystem, are correctly defined. Thus, in this research, two different formulations are implemented in the subsystem module to validate the proposed method. One is the analytical method, in which trigonometric functions are used in position, velocity and acceleration analyses. The other one is the numerical method in which Newton-Raphson method is used for position analysis and Gaussian elimination method is used for velocity and acceleration analyses.

Fig. 9 shows the one subsystem of the excavator for the analytical method.

θ_1 and θ_2 can be calculated by using the cosine law in the triangle geometry as shown in Fig. 9.

$$\theta_1=\cos^{-1}\left(\frac{l_1^2+l_2^2-l_t^2}{2l_1l_2}\right),\quad \theta_2=\cos^{-1}\left(\frac{l_2^2+l_t^2-l_1^2}{2l_2l_t}\right) \tag{5}$$

where, $l_t=l_3+l_4+q_t$. Joint coordinates q_r and q_{u1} can be obtained from θ_1 and θ_2 in Eq. (5).

Velocity and acceleration analysis can be accomplished by using Eq. (6) and Eq. (7), respectively.

$$\begin{aligned}\dot{\theta}_1&=\left(\frac{l_t}{l_1l_2\sin(\theta_1)}\right)\dot{q}_t=N_1\dot{q}_t\\ \dot{\theta}_2&=\left(\frac{l_2^2-l_1^2-l_t^2}{2l_2l_t^2\sin(\theta_2)}\right)\dot{q}_t=N_2\dot{q}_t\end{aligned} \tag{6}$$

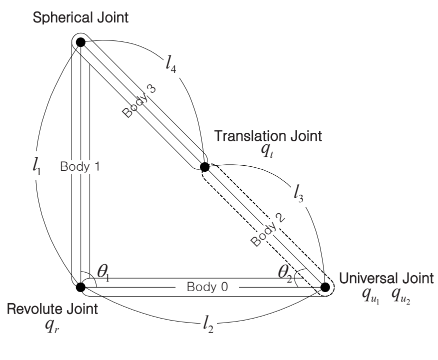

Fig. 9. One subsystem of excavator.

Table 1. Error of analysis.

	Numerical	Analytical
Position	10^{-4} m order	10^{-3} m order
Velocity	10^{-3} m/s order	10^{-3} m/s order
Acceleration	10^{-2} m/s^2 order	10^{-2} m/s^2 order

Table 2. CPU time of analysis.

	Numerical	Analytical
Position	5.115 s	3.186 s
Velocity	1.974 s	1.624 s
Acceleration	2.66 s	2.673 s
Total CPU	9.749 s	7.483 s

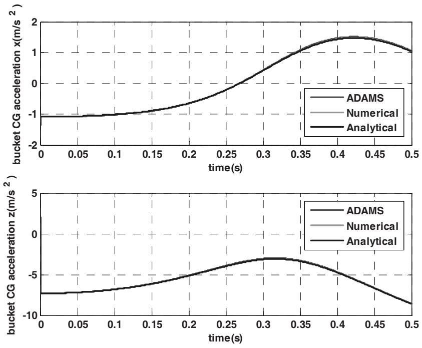

Fig. 10. X and Z acceleration of bucket's CG point.

$$\begin{aligned} \ddot{\theta}_1 &= N_1\ddot{q}_t + \dot{N}_1\dot{q}_t \\ \ddot{\theta}_2 &= N_2\ddot{q}_t + \dot{N}_2\dot{q}_t \end{aligned} \tag{7}$$

3.3 Simulation results

Simulation results from the recursive subsystem synthesis method with the analytical analysis and with the numerical analysis are compared. ADAMS solution is treated as the reference solution. Essentially, the identical results are obtained, compared with the ADAMS solutions as shown in Fig. 10. Table 1 also shows the analysis error. These results show that the proposed method provides accurate solutions.

Efficiency is also compared from these two different methods by measuring CPU time with Matlab software. Table 2 shows the CPU time result. A personal computer has been used with Intel Core2Quad 2.4GHz CPU and 2GB memory. As shown in Table 2, the analytical method is more efficient, since the Newton-Raphson iteration was avoided.

4. Conclusions

The recursive subsystem synthesis method has been developed for a repeated closed loop structure. The proposed method has been validated by comparing solutions with those from the ADAMS model. Two different methods within the subsystem were implemented for kinematical analysis using Matlab software. One is analytical method and the other is numerical method. Both methods produced the same results as expected. However, the analytical method provides a little bit better efficiency.

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